Credibility, Public Debt and
Foreign Exchange Intervention

Mo Chaudhury and K. Ramagopal*

This Draft: July 20, 2001

Abstract

This paper presents a generalized framework where a discretionary
government finds it ex-post optimal to create a downward bias in the
exchange rate in addition to the inflation bias of Barro-Gordon type. This
dual credibility model helps explain the empirical pattern of public debt
financing most recently reported by Falcetti and Missale (2000). Most
notably, large and developed economies are predicted to have lower
inflation and more conventional home currency bonds. The small and
emerging economies, on the other hand, are expected to have relatively
higher inflation and more foreign currency bonds. This suggests that a
judicious use of a portfolio of home and foreign currency bonds may help
economies that are in transition to independent central banking.

Keywords: credibility, public debt, foreign exchange intervention.
JEL Classification: E52, E58, F41, H63.

Corresponding Address: Mo Chaudhury, Faculty of Management, McGill University, 1001 Sherbrooke
Street West, Montreal, Quebec, Canada H3A 1G5. Tel: (514) 398-5927, Fax: (514) 398-3876, Email:
chaudhur@management.mcgill.ca.

* Chaudhury is from the Faculty of Management, McGill University, 1001 Sherbrooke Street
West, Montreal, Quebec, Canada H3A1G5, Email: chaudhur@management.mcgill.ca; and
Ramagopal is with Financial Research, Freddie Mac, 8200 Jones Brand Drive, Mail Stop 448,
McLean, VA 22102-3110, USA, Email: buchi_ramagopal@freddiemac.com.
1. Introduction

Since the seminal work of Kydland and Prescott (1977) and Barro and Gordon (1983a, 1983b), it is well known that governments following a discretionary monetary policy through a dependent central bank cannot credibly commit to an inflation target. Inflation surprises expand output and reduce real taxes by reducing the real wage and the real value of nominally fixed home currency bonds (H-Bonds). Therefore, once agents enter into nominal wage contracts and purchase H-Bonds, it is ex-post optimal for a social welfare maximizing government to deviate from the ex-ante (optimal) inflation target. This has come to be known as the inflation credibility problem or alternatively as the dynamic time inconsistency problem.\(^1\)

One of the solutions suggested in the literature (Bohn (1990a, 1991), Watanabe (1992), Miller (1997)) is the issuance of foreign currency denominated bonds (F-bonds). If the Relative Purchasing Power Parity (RPPP) holds on average, the issuance of F-Bonds should eliminate the financing-related inflation credibility problem. In this paper, we generalize the economic setup to allow for foreign exchange interventions by the government when it uses F-Bonds. This leads to an exchange rate credibility problem in addition to the inflation credibility problem. Within this dual credibility framework, we then explore the nature of optimal inflation and exchange rate biases and provide a theoretical explanation for the empirically observed pattern of government financing (Falcetti and Missale (2000)).

The design of debt contracts to address the inflation credibility problem has produced three sets of suggested solutions. The first considers the management of debt maturity. Missale and Blanchard (1994) make the case that governments with a large volume of outstanding debt should use short-term debt the value of which is not sensitive to inflation surprises and this reduces the benefits of deviating from an inflation pledge.\(^2\) The second considers the issuance of inflation-indexed bonds. According to Calvo and Guidotti (1990) as well as Persson, Persson and Svensson (1987), these bonds neutralize

---

\(^1\) See Fischer (1990), Chapter 9 of Obstfeld and Rogoff (1998) and Chapter 8 of Walsh (1998) for a detailed discussion of the rules versus discretion debate.

\(^2\) In support of their argument, Missale and Blanchard (1994) report that for three highly indebted countries, Belgium, Ireland and Italy, the share of fixed rate long-term debt exhibited a negative relation with the debt-to-GDP ratio during the 1960-1989 period. In a similar vein, Campbell (1995) argues that the normally observed positive interest rate spread of the long-term bonds over the short-term bonds may reflect the credibility...
government’s incentive to engineer inflationary surprises and thus improve social welfare. The final solution (Bohn (1990a, 1991), Watanabe (1992), Miller (1997)) to the debt-related credibility problem is the issuance of F-Bonds (F-Bond). F-Bonds has a built-in commitment device. Pursuant to the issuance of F-Bonds, should the government resort to engineering inflation, it pays a penalty because the home currency depreciates according to the Relative Purchasing Power Parity.\footnote{Further, this scheme has the added advantage that should the foreign government inflate, the domestic government receives a windfall as the home currency appreciates and the home currency value of the outstanding debt is lower. To finance employment-enhancing expenditures and/or reduce taxes, the home government can use this windfall gain instead of creating inflationary surprises.}

Despite the inflation-commitment benefits of short maturity (or longer maturity floating rate) bonds and inflation-indexed bonds, empirical evidence shows that governments rarely use them. In a recent study, Falcetti and Missale (2000) report that conventional longer-term (maturity more than one year) fixed rate bonds remain as prevalent as ever.\footnote{Further, this scheme has the added advantage that should the foreign government inflate, the domestic government receives a windfall as the home currency depreciates according to the Relative Purchasing Power Parity.} Further, the empirical evidence of Falcetti and Missale (2000) regarding twenty OECD countries during the 1970-1997 period show that on average only 16% of the debt was denominated in foreign currency despite their built-in inflation commitment mechanism.

The use of F-Bonds, however, varies widely across countries. Smaller economies with underdeveloped fixed income markets and those characterized by fears of inflation and substantial currency depreciation use F-Bond more heavily. For example, the share of F-Bond is relatively high in the range of 25% to 35% in Austria, Denmark, Greece, Ireland and New Zealand with Finland topping the list with 46%. At the other extreme, countries with central banks known for their strong anti-inflationary stance (for example, Germany, Japan, the Netherlands and Switzerland) and/or who enjoy lower borrowing cost as their currencies are used as foreign reserves (for example, US), virtually does not use any F-Bond. The time trend shows the use of F-Bonds peaked at a cross-country average of 23% in 1985 and has declined steadily since then to an average of 15% in 1997. The same pattern is observed for most individual countries as well. Interestingly, the declining use of F-Bond coincides with increased independence of the central banks in many countries.

Researchers have advanced a number of arguments to explain the empirical pattern of government financing. The arguments to explain the preference of long maturity fixed
rate home currency bonds over short maturity bonds and inflation-linked bonds include tax smoothing (Barro (1995, 1997), Bohn (1988, 1990b), Lucas and Stokey (1983), Calvo and Guidotti (1990)), trading frictions and portfolio hedging considerations (Fischer (1983)), reduced roll-over risk (Calvo (1988), Alesina, Prati and Tabellini (1990), Giavazzi and Pagano (1990), Sachs, Tornell and Velasco (1996)), and enhanced chances of successful fiscal stabilization program (Missale, Giavazzi and Benigno (2000). However, these justifications do not quite explain why most governments use a portfolio of home currency and foreign currency bonds and the way this portfolio mix varies across countries and over time.

Building on previous works (Rogoff (1985), Walsh (1995)) on the design of monetary institutions to address the inflation credibility issue, most recently Falcetti and Missale (2000) model the interaction between institutional design and debt design as a tradeoff between monetary policy credibility and output flexibility. In their model, by issuing F-Bonds or inflation-indexed bonds to handle the inflation credibility problem, a government is giving up an important source of inflation tax to pursue output goals. Falcetti and Missale (FM) show that a greater use of long-term fixed rate home currency debt enhances the sensitivity of output to surprise inflation and thus preserves flexibility without the need for large inflation surprises. This can be achieved without any significant inflation credibility problem if the monetary policy is conducted independently by a highly conservative central bank.

FM’s model does explain the preference for conventional long-term fixed rate debt and the global trend towards independent monetary institutions. However, several questions or concerns remain unresolved. First, FM’s model suggests maximum permissible use of long-term fixed rate debt. This may not sound attractive to many governments and the increasingly conservative central and supranational bankers as the socially acceptable limits to public debt are falling across the world driven by preference for fiscal prudence and intergenerational considerations. For small economies and countries with low savings rates, too much funding in home currency bonds may not be feasible either.

Second, FM’s model suggests minimal to no use of foreign currency denominated bonds. While empirically the use of F-Bonds and the independence of monetary institution

---

4 Missale, Giavazzi and Benigno (2000) provide evidence that the OECD countries actively manage the share of fixed-rate long-term debt depending on the level and conditional volatility of the interest rates.

5 Relatively conservative here means that the central banker has a stronger aversion to inflation than the society at large. The government is assumed to reflect the society’s preferences.
may be (negatively) related, the linkage is not decisive theoretically. The reason FM’s model suggests minimum possible use of F-bonds is that in their model these bonds do not provide any flexibility benefit like the inflation tax of H-Bonds. The inflation credibility benefit of F-bonds is, on the other hand, maintained through the independent monetary institution. As it will be shown in the generalized dual credibility model of this paper, F-Bonds actually yield flexibility benefit similar to H-Bonds when foreign exchange intervention is permitted. Thus, it is not clear whether the independence of monetary institution can satisfactorily explain the continued use of F-Bonds and the cross-sectional variation in their use.

Third, for smaller, emerging and less democratic countries, an independent monetary institution may not exist. Even when such an institution formally exists, the independence of the institution may be questionable. Until the transition to a truly independent monetary institution is complete in these economies, foreign currency denominated bonds is likely to play an important role in keeping inflation temptation in check. The optimal inflation and exchange rate intervention policies and the mix of home and foreign currency bonds then need to be addressed.

Lastly, in trying to solve the credibility-flexibility tradeoff, the highly conservative independent central banking institution in FM’s model in fact creates some rigidity in terms of implementing and dynamically adjusting the solution. In general it is easier and less expensive to issue and adjust a portfolio of home and foreign currency bonds than to put in place an institution and adjust the nature of this institution if and when needed. Therefore, until the small and emerging economies become mature, the portfolio of bonds approach may provide useful guidance for policy optimization.

The portfolio solution has its limitations too. The beneficial role of foreign currency bonds in restraining inflation temptation depends on the RPPP. To the extent the government is able and willing to intervene in the foreign exchange market, the inflation credibility benefits of F-bonds may be at risk, but a second source of discretionary real financing (and output flexibility) similar to the inflation tax is created. A key contribution of this paper is the modeling of this extended tradeoff.

The predictions of our generalized dual credibility model are consistent with the empirically observed pattern of government financing. When the terms of the financing instruments (bonds) are not optimized at the time of issuance, our model predicts that the inflation bias will be relatively smaller than the foreign exchange bias if the society is more
concerned about the costs of inflation than the costs of foreign exchange bias. In terms of market value, this will enhance the proportion of home currency bond financing. When the government optimizes the terms of the bonds, the society’s relative preference for the costs of inflation and foreign exchange biases create a similar home currency bias in the optimal mix of home and foreign currency bonds. These predicted patterns are typically found in the large and developed economies. Using the same argument, our model predicts a relatively higher inflation bias and a lesser home currency bias in the small and emerging economies where it is easier for the government to intervene in the foreign exchange market and the society is more concerned about the foreign exchange bias.

In our model, the level of the inflation and foreign exchange biases are expected to be higher if the marginal output benefit from an inflationary surprise is higher and/or the society values more the real financing benefit (via lower real taxes) from unanticipated inflation and foreign exchange intervention. In other words, the policy biases increase if the flexibility benefits of discretion are greater relative to the credibility costs. Not only this captures the essence of the flexibility versus credibility tradeoff of discretion, it is also consistent with the fact that the small and emerging economies usually have higher inflation and the governments in these economies are more active in managing the exchange rate. Compared to the large and developed economies, the small and emerging economies have of course a much lower capital base and their citizens are typically less tolerant of taxes and are taxed less.

In addition to explaining the cross-sectional pattern of policy biases and financing mix, our generalized dual credibility problem also successfully predicts lower levels of policy (inflation and foreign exchange) biases over time as citizens worldwide became increasingly aware and concerned about the social costs of government discretion. Given the unpleasant inflationary experience of the seventies and eighties, the importance of the social costs of inflation (relative to the flexibility benefits) has likely increased more compared to the importance of the costs of foreign exchange bias. In light of our model, this should help explain the gradual shift towards home currency financing and away from foreign currency financing, as reported by Falcetti and Missale (2000).

The rest of the paper is organized as follows. In section 2, we describe the economic setup within which the classic inflation credibility problem arises. We also discuss here the role of the F-Bonds in resolving the inflation credibility problem as suggested in the
literature. The economic setup is generalized in Section 3 to allow foreign exchange interventions by the government. To keep our focus on the credibility issue, we only entertain a deterministic (negative) effect of interventions on the drift of the exchange rate and ignore any effect on the (conditional) volatility.\textsuperscript{6} Until this point, we take the terms of the government’s financing package as given. In section 4, we further allow the optimization of bond terms, more specifically the face value of home currency bonds, leading to an optimal mix of home and foreign currency denominated bonds. Concluding comments follow in section 5.

2. The classic inflation credibility problem

We first lay out the economic setup within which the classic inflation credibility problem arises. There are four components of our basic economic setup, viz., the macroeconomic environment, the debt financing setup, the bond pricing framework, and the optimization framework for the government. We shall also introduce the exchange rate process later when we consider F-Bonds. We assume risk neutral agents and government throughout the paper.

2.1 The macroeconomic environment

There are two countries, home (small) and foreign (large). Since the foreign country is large and we do not consider policy coordination or other forms of interaction between the home and foreign governments, foreign inflation ($\pi^*$), nominal ($r^*$) and real ($\rho^*$) interest rates are treated as exogenous. We also assume the foreign variables to be non-stochastic for simplicity.

The macroeconomic setup for the home country is of Barro-Gordon type. There is no separation between the monetary authority and the government. The government does not follow any rule and instead exercises discretion with respect to monetary and fiscal actions. At the beginning of time 0, agents form rational expectation of a constant inflation

\textsuperscript{6} See Dominguez (1998) for conditional volatility effect of exchange rate interventions.
rate ($\pi^n$) per period for the next $T$ periods and enter into $T$-period employment contracts at a nominally fixed wage rate.\footnote{The length of one period may be such that that $T$ periods is long enough for policy purpose but not too long for wage contracts.} The government then decides upon the actual inflation rate ($\pi$) and executes this rate during the $T$ periods.\footnote{We assume that the home government can control the inflation rate without any lag or disturbance by controlling the money supply growth. With velocity disturbance, the government controls the mean inflation rate. Also, we are ignoring a possible zero-mean supply shock that the government observes (but the agents do not) after the agents have formed expectation but before the government decides mean inflation rate. See Walsh (1998), Chapter 8, for an excellent exposition of these and other related issues.} Based on the nominally fixed wage rate and the inflation rate chosen by the government, profit maximizing producers then determine the employment level leading to an aggregate log output, $y$, per period. This results in a Lucas-type aggregate supply function or expectation-augmented Phillips curve: $y - y^0 = \alpha_0 (\pi - \pi^n)$, where $y^0$ is the government’s output target and it is equal to the log natural rate of output plus any upward adjustment to reflect output lost due to frictions.\footnote{The parameter $\alpha_0$ is increasing in real labor productivity in a Cobb-Douglas production function.}

2.2 The debt financing setup

At time 0, the home government inherits an exogenous nominal debt of $D_0$ in home currency. To refinance this debt, initially we assume that the government sells $T$-period zero-coupon nominal home currency bonds, called H-Bonds. The face value of an H-Bond is one unit of home currency\footnote{The length of one period may be such that that $T$ periods is long enough for policy purpose but not too long for wage contracts.} and its time 0 price is denoted by $B_{H,0}$. To refinance $D_0$, the government needs to issue $D_0/B_{H,0}$ units of H-Bonds. The time $T$ nominal payment obligation of the government will be $D_0/B_{H,0}$ all of it in home currency.

2.3 The bond pricing framework

Since our focus in this paper is on the credibility issues, we assume that there is no uncertainty about the instantaneous home nominal riskfree rate, $r(t)$. This rate is determined according to the Fisher equation:

$$r(t) = \rho(t) + \pi^n(t)$$

(1)

As the government executes the chosen inflation rate, $\pi$, throughout the $T$ periods, the rationally anticipated inflation rate will be a constant, $\pi^n$. With the home real rate, $\rho(t)$,
assumed equal to the constant foreign real rate, $\rho^*$, the home instantaneous nominal rate, $r(t)$, maintains a constant level.\(^{11}\)

\[
r(t) = r = \rho^* + \pi^e
\]  

(2)

Investors buy the Bonds at time 0 before the government decides $\pi$. The home currency nominal price paid by the investors for an H-Bond and the associated real taxes at $T$ are respectively:

\[
B_{H,0} = \exp(-r_{H,T}) = \exp[-(\rho^* + \pi^e)T] \tag{3}
\]

\[
\tau_{H,T} = \left(\frac{D_0}{P_0}\right) \exp \left[(\rho^* + \pi^e - \pi)T\right] \tag{4}
\]

The H-subscript for the expected inflation and the interest rate indicates the levels of these variables when the government issues H-Bonds, while $P_0$ represents the time 0 price level.

2.4 The optimization framework for the government

We are using the dependent central bank framework along the line of Missale and Blanchard (1994). However, we use one-shot game instead of multiperiod reputational equilibrium. Also, we deal with T-period contracts instead of rolling over one-period contracts. As we shall see, the essential nature of the credibility problem is retained in our simplified framework.

The sequence of events leading to the government’s optimization problem is described now. At the beginning of time 0, the government announces the type of T-period obligation it is using to refinance the inherited nominal debt of $D_0$. Economic agents form rational expectations about inflation ($\pi^e$). Based on these expectations, the agents enter into T-period fixed nominal wage contracts and buy the T-period government obligations at a nominal price reflecting $\pi^e$.

The government then chooses the actual level of $\pi$ by maximizing the following social welfare function:\(^{12}\)

\[
U = \alpha(\pi - \pi^e) - 0.5 \delta \pi^2 - w \ln[\tau_{H,T}] \tag{5}
\]

\[^{10}\] Later, we shall generalize the face value to K units of home currency.

\(^{11}\] Combined with no default or liquidity risk, this results in a flat nominal term structure.

\(^{12}\] This type of welfare function is standard in the credibility literature. See Walsh (1998).
where \( \alpha = \alpha_0 \{1-\exp[-\beta T]\}/\beta \), \( \delta = \delta_0 \{1-\exp[-\beta T]\}/\beta \), \( w = w_0 \exp[-\beta T] \), and \( \beta \) is the subjective discount rate of the society. The term \( \alpha(\pi - \pi^* e^H) \) represents the discounted value of the stream of output gains from the inflation surprise, \( \pi - \pi^* e^H \), per period from 0 to \( T \).\(^{13}\) Obviously, \( \pi - \pi^* e^H \) won’t be a surprise in a literal sense after the first period. However, in our framework, \( \pi - \pi^* e^H \) effectively remains as a surprise since this level of unanticipated inflation is sustained by the government every period until time \( T \) and the economic agents have already committed to \( T \)-period contracts.

In keeping with the literature, we assume that the costs of inflation as seen by the society are quadratic in the actual rates of inflation.\(^{14}\) Also, the society views zero inflation as desirable. The weights, \( \delta_0 \) and \( w_0 \), represent social welfare importance of the costs of inflation and taxes relative to the benefits of increased output.

The government’s initial budget constraint is that the proceeds from issuing new \( T \)-period obligation equal its outstanding debt amount \( D_0 \). Since the government expenditures play no material role and our focus is on debt financing, we assume along the line of Missale and Blanchard (1994) that at time \( T \) the government raises real tax revenue, \( \tau_{H,T} \), sufficient to meet the real cashflow or payment obligation required of it.\(^{15}\) We assume that the costs of real taxes are linear in log of real taxes.\(^{16}\)

Thus, in essence, we have a one-period (of length \( T \)) framework for the government’s optimization problem. Also, the bond pricing framework and the assumed utility function are consistent with risk neutral agents and government.

### 2.5. The inflation credibility problem

From the first-order condition, the government’s optimal choice of inflation rate is given by:

\(^{13}\) Similar to Barro and Gordon (1983b) and Missale and Blanchard (1994), we are assuming that the society cares about output expansion rather than output stabilization. The implication is that the welfare function is then linear in output.

\(^{14}\) See Driffill, Mizon and Ulph (1990) for the costs of inflation.

\(^{15}\) There is no intertemporal tax smoothing or tax rate stabilization in our model. But there is an implicit inverse relationship between output and the time \( T \) tax rate. In bad states the lump sum taxation needed to payoff the financing obligations will translate to a higher tax rate at time \( T \).

\(^{16}\) Given the log form, the social welfare cost is increasing but concave in real taxes. The inflation costs, on the other hand, are convex (quadratic) in the inflation rate. However, this difference is not important in our model as we exclude higher order moments of real taxes in our approximation.
\[ \pi^H = \frac{(\alpha + wT)}{\delta} \]  

Rational agents expect this choice by the government and hence use \( \pi^{\text{eH}} = \frac{(\alpha + wT)}{\delta} \) in their consideration of the fixed nominal wage rate and the price they pay for the H-Bonds. The government's actual choice of inflation rate is then forced to this non-zero rate of inflation. This is the classic inflation bias that arises because the government cannot credibly commit to a zero rate of inflation. Economic agents foresee that even if the government announces a zero inflation target before the determination of the fixed nominal wage rate and the bond price, it will be \( \text{ex-post} \) optimal for the social welfare maximizing government to deviate from that target and create a positive rate of inflation.

The financial cost of this inflation credibility problem is that the nominal cost of borrowing, \( r = \rho^* + \pi^{\text{eH}} \), is higher than the real rate \( \rho^* \) with \( \pi^{\text{eH}} \) reflecting the inflation credibility premium that the economic agents charge. Although the real cost of borrowing remains at \( \rho^* \), social welfare is lessened by \( 0.5\delta[(\alpha+wT)/\delta]^2 \) due to lack of a credible commitment mechanism.

If the expected real tax at time T is prorated over the whole period, the structure of the weight parameters, \( \alpha \), \( \delta \), and \( w \), in terms of T is the same. The optimal inflation rate, \( \pi^H=(\alpha+wT)/\delta \), is then increasing in T. Thus the choice of a shorter maturity bond reduces the inflation bias as in Missale and Blanchard (1994).

### 2.6 The institutional solution

The institutional solution to the inflation credibility problem is a weight-conservative independent central banker (Rogoff (1985)).\(^{17}\) Such a central banker would presumably exhibit greater distaste for inflation (a higher \( \delta_0 \)) than the government and the society at large. The independent central banker would still be inclined to engineer inflation surprise. However, the magnitude of inflation chosen would be lower than if the monetary policy were to be conducted by a dependent central banker.

---

\(^{17}\) The alternative institutional solution is to offer a compensation scheme to the central banker whereby the central banker's compensation decreases with higher inflation. The central banker's objective function will reflect this incentive aspect of a lower inflation rate and hence will result in a moderated rate of inflation. See Walsh (1998), pp. 355-360.
2.7 The foreign currency bond solution

To see how the use of foreign currency denominated bonds (F-Bonds) may reduce the inflation bias, suppose now that the government issues T-period zero-coupon F-Bonds only. The face value of a F-Bond is one unit of foreign currency and its time 0 price in foreign currency is denoted by $B_{F,0} = \exp[-r^F T]$, where $r^F$ is the nominal interest rate on the F-Bonds. The determination of $r^F$ will be discussed shortly following the specification of the exchange rate process.

Given $S_0$, the exchange rate (home currency per unit of foreign currency) at time 0, the time price of a F-Bond in home currency is $S_0 B_{F,0}$. To refinance $D_0$, the government needs to issue $D_0/(S_0 B_{F,0})$ units of F-Bonds. The time T nominal payment obligation of the government will be $D_0/(S_0 B_{F,0})$, all of it in foreign currency. The home currency equivalent of this obligation will be $D_0 S_T/(S_0 B_{F,0})$, where $S_T$ is the time T exchange rate.

To start with, we assume a fully floating exchange rate regime. While there is an extensive empirical literature on the exchange rate process, we adopt here the most commonly used Exponential Martingale or Geometric Brownian Motion model for the nominal exchange rate, $S$:

$$dS/S = \eta \, dt + \sigma \, dW$$

where $dW$ is the increment of a Weiner Process, $\eta$ and $\sigma$ are respectively the instantaneous drift and volatility of geometric return on holding the foreign currency. The exchange rate model above can be represented in an alternative but equivalent way:

$$S_t = S_0 \exp(\mu t + \sigma W_t)$$

where $W_0=0$, $W_t \sim \mathcal{N}(0, \sigma^2 t)$, and $\mu = \eta - 0.5 \sigma^2$ is the conditional mean logarithmic return on foreign currency per unit of time. Stated differently, exchange rate is conditionally lognormally distributed, i.e., $\ln(S_t/S_0) \sim \mathcal{N}(\mu t, \sigma^2 t)$.

The mean logarithmic return, $\mu$, is determined by the economic fundamentals (see, e.g., Lewis (1995)). We assume that the Relative Purchasing Power Parity (RPP) holds on average barring foreign exchange interventions and accordingly $\mu$ equals the inflation differential, $\pi - \pi^*$, less the lognormal adjustment $0.5 \sigma^2$. As we assume risk neutrality, the forward rate, $F_0$, equals the spot rate expected by the market participants or agents:

$$F_0 = E_M(S_T) = S_0 \exp[(\pi^F - \pi^*) T]$$
With risk neutrality or equivalently covered or uncovered interest rate parity, we have the time 0 price of a foreign currency denominated bond in home currency as:

\[ S_0 B_{F,0} = \exp[-rT] E_M(S_T) = S_0 \exp[-(\rho^* + \pi^*)T] \] (10)

Hence, barring foreign exchange interventions, the nominal interest rate on the F-Bonds and the price of a F-Bond are respectively the international equivalents:

\[ r^F = \rho^* + \pi^* \] (11)

\[ B_{F,0} = \exp[-r^*T] = \exp[-(\rho^* + \pi^*)T] \] (12)

It is important to recognize that the market prices of securities (and therefore the nominal interest rates too) are determined according to the rational expectations of the agents. These expectations of course depend on how the government refines its existing debt. However, it is the actual inflation rate that the government optimally chooses that determine the actual drift of the exchange rate process. Therefore, the expected real taxes that the government considers in its optimization depend on the exchange rate drift as projected by the government:

\[ E_G(S_T) = S_0 \exp[(\pi - \pi^*)T] \] (13)

Accordingly, for a given choice of the actual inflation rate, \( \pi \), following the sale of the bonds at the price, \( \exp[-(\rho^* + \pi^*)T] \), the government expects its time T real taxes to be:

\[ E_G(\tau_{FT}) = (D_0/P_0) \exp[\rho^*T] \] (14)

The government then chooses the actual level of \( \pi \) by maximizing the social welfare function:

\[ U = \alpha(\pi - \pi^G) - 0.5 \delta \pi^2 - w \ln E_G(\tau_{FT}) \] (15)

Due to exchange rate contingency of the obligation, the government considers the expected costs of real taxes in its optimization. We use the weighted log of expected real taxes, \( w \ln E_G(\tau_{FT}) \), as an approximation to the expected costs of real taxes. From the first-order condition, the government’s optimal choice of inflation rate is then given by:

---

18 We do not consider sovereign default risk or secondary market liquidity risk here. In general, there will be corresponding risk premiums on the F-Bonds of small and emerging economies.
19 For the exchange rate, the rational expectation in addition depends on whether the government intervenes in the foreign exchange market.
20 Possible foreign exchange intervention effects will be discussed shortly.
21 The expectation is with respect to the probability distribution of the exchange rate at time T. The government does not know this exchange rate when it optimally sets the inflation rate and the foreign exchange bias at time 0 given the rational expectation of the economic agents. In the credibility literature, the traditional
Thus when there is no foreign exchange intervention and the government solely uses F-Bonds, the inflation bias pertaining to debt financing is eliminated. The foundation of this F-Bond solution is that any inflation surprise leads to an equal expected appreciation of the exchange rate (via parity). Accordingly, the nominal home currency value of F-Bonds goes up enough to offset the depreciation caused by an inflation surprise and as such there is no inflation temptation to reduce the real taxes needed to pay off the F-Bonds.

2.8 The portfolio of bonds solution

The reduced inflation bias associated with the F-Bonds, however, comes at the cost of flexibility (Falcetti and Missale (2000)). With inflation having no impact on the real value of the F-bonds, the government has lost a potential source of real financing and fiscal stimulus. Hence the government needs to generate a greater inflation surprise to achieve an output target or to counteract a negative supply shock.

It then follows that to achieve a desired tradeoff between credibility and flexibility, the optimal financial solution would involve issuing a portfolio of H-Bonds and F-Bonds. Suppose the government sells these bonds as packages of one H-Bond and one F-Bond per package. Each H-Bond has a face value of $K$ units of home currency while as before each F-Bond has a face value of one unit of foreign currency. Assume that the government will not intervene in the foreign exchange market.

The (government’s) expected time $T$ exchange rate, the proportion of financing in terms of H-Bonds and the (government’s) expected time $T$ real taxes are respectively:

$$E_G(S_T) = S_0 \exp[(\pi - \pi^*)T]$$  \hspace{1cm} (17)

$$m_p = \frac{K}{K + S_0 \exp[(\pi^p - \pi^*)T]}$$  \hspace{1cm} (18)

Assumption is that the government observes the supply shock before it optimally sets the inflation rate. Since we consider output expansion rather than stabilization, we omit explicit modeling of the supply shock.

22 That is, in taking expectation of the Taylor series expansion for log real taxes around the expected real taxes, we ignore the second and higher order terms. Thus, we are assuming that the society cares more about lowering real taxes than about stabilizing real taxes. Alternatively, the effect of the variance and higher order moments of the real taxes is likely small. This is because we do not explicitly model stochastic variations in inflation and output. The real taxes are uncertain in our model only due to other sources of stochastic variations in the exchange rate and possible exchange rate contingencies of the obligations.

23 Here we do not explicitly model supply shocks and government expenditures. See Missale and Blanchard (1994) and Falcetti and Missale (2000).
$$E_G(\tau_{PT}) = \left(\frac{m_T}{K}\right) \left(D_0/P_0\right) [K+E_G(S_T)] \exp[\rho^* + \pi^e - \pi]T$$

(19)

Note that while the government can attempt to increase its home currency financing by selecting a higher $K$, the financing proportion $m_T$ is ultimately determined (via bond pricing) by rational expectation of inflation that in turn depends on the solution to the government’s optimization problem.

As before, the government chooses the actual level of $\pi$ by maximizing the social welfare function:

$$U = \alpha(\pi - \pi^0) - 0.5 \delta \pi^2 - w \ln E_G(\tau_{PT})$$

(20)

The first order condition for this optimization is:

$$\alpha - \delta \pi + w T m_T = 0$$

(21)

where $m_T = K/\{K + S_0 \exp[(\pi - \pi^0)T]\}$ and it represents the proportion of expected real taxes that the government needs to pay off the H-Bonds. In the above condition, $\delta \pi$ is the marginal (welfare) cost of an inflation while $(\alpha + w T m_T)$ is the associated marginal benefit. Thus, the financing related component of the marginal benefit of inflation is due to the reduction in expected real taxes required to pay off the H-Bonds and the consequent expected availability of real financing. The expected real taxes for the foreign currency denominated portion of financing are not affected by inflation as the depreciation of the home currency cancels out the inflation effect by virtue of the Relative Purchasing Power Parity. Thus the foreign currency denominated component of the financing package helps contain the inflation bias and serves the purpose of a conservative independent central banker. The expected real financing availability due to the inflation-sensitive home currency denominated component, on the other hand, provides the flexibility discretionary governments may desire on behalf of the society. As noted previously in the literature (e.g., Missale and Blanchard (1994)), the marginal real financing benefit of inflation is greater if the government uses longer maturity bonds (a higher $T$).

While the credibility versus flexibility result is not new, we state it below in the form of a proposition given its importance and for the sake of completeness in the context of our $T$-period economic setup that is slightly modified from the literature.
Proposition 1:
In a dependent central bank framework, if the government uses a portfolio of home currency and foreign currency bonds, entertains only inflation surprise and the terms of bonds are given, then a tradeoff between inflation credibility and output flexibility is created.

There is no closed form analytic expression for the optimal choice of $\pi$. However, that $\pi$ will satisfy $\alpha / \delta \leq \pi \leq (\alpha + wT) / \delta$ can be seen from the first-order condition above. Regardless of the choice of $\pi$, the proportion of expected real taxes on account of the H-Bonds, $m_\tau$, lies in the closed interval $[0,1]$. Therefore, the marginal benefit of an inflation surprise is reduced when using a portfolio of bonds as compared to using H-Bonds alone. This results in a reduced inflation bias ($\pi^p < \pi^H$) and a lower nominal cost of borrowing in home currency. However, the inflation bias remains higher compared to the case of using F-Bonds alone ($\pi^F > \pi^p$).

Note that a zero rate of inflation can be (optimally) achieved and thus the inflation bias can be totally eliminated if the government selects $K < 0$.

Corollary 1:
In a dependent central bank framework when the government entertains an inflation bias ($\pi$) but no foreign exchange bias and uses a financing package of one K face value home currency bond and one unit face value foreign currency bond, then a zero rate of inflation can be achieved by setting:

$$K = - \left[ \alpha / (\alpha + wT) \right] S_0 \exp[- \pi T]$$

Proof: Inserting the above value of $K$ in the first order condition for the inflation rate, the condition reduces to $\delta \pi = 0$ and hence the solution $\pi = 0$ obtains.

---

24 In finance parlance, time to maturity $T$ is the duration of a zero coupon bond. To a first order, duration measures the sensitivity of bond value with respect to the interest rate. Bonds with longer duration are more sensitive to interest rate shocks.
This zero inflation choice means issuing more F-Bonds than is needed to meet the financing need and then lend the excess in home currency to the private sector. This type of government financing is of course quite unrealistic and is uncommon too. In light of real life experience, tradeoff solutions with $0 < m_c < 1$ seem more appropriate. Further, as we shall see shortly, when the government can effectively intervene in the foreign exchange market, a zero inflation financing policy can prove to be worse to the society than using H-Bonds alone (and experience maximum inflation bias).

3. The foreign exchange credibility problem

In this section, first, we will discuss how an exchange rate credibility problem arises in addition to the inflation credibility problem when the government uses F-Bonds alone. Second, the dual (inflation and exchange rate) credibility problem in the context of a portfolio of bonds will be examined. Lastly, the desirability of the zero inflation policy will be evaluated.

3.1 F-Bonds and the exchange rate credibility problem

Suppose the government issues F-Bonds only to refinance its existing debt. Assuming no default risk, the payment obligation for the F-Bond is fixed in nominal foreign currency terms. Hence the nominal cash outflow required of the home government at $T$, i.e., $S_T$ units of home currency, remains exposed to $S_T$. To the extent $S_T$ is lower, the nominal obligation of the government in home currency is reduced. Prior to maturity, a lower $S_T$, i.e., a depreciation (appreciation) of the foreign (home) currency, will translate to a lower home currency value of the F-Bond. For any given rate of inflation, a depreciation of the foreign currency thus constitutes a source of real financing to the government similar to an inflation surprise or seigniorage. To the extent it is possible to influence the exchange rate, the government has an incentive to intervene in the foreign exchange market and reduce the exchange rate once economic agents have formed their rational expectation of the exchange rate movements. As such the government cannot credibly commit to a no-intervention

---

25 See Falcetti and Missale (2000) for this result derived from a somewhat different formulation.
policy if it issues foreign exchange obligations and it is well known that the government interventions can influence the exchange rate.

Rational buyers will factor this exchange rate credibility issue into pricing the F-Bond or other exchange rate contingent obligations. This in turn would drive up the cost of borrowing for the government and the government will be forced to fulfill the rationally anticipated foreign exchange interventions. Thus the issuance of F-Bonds replaces the inflation credibility problem pertaining to home currency financing with an equivalent exchange rate credibility problem.

With the possibility of foreign exchange interventions, the actual evolution of the exchange rate will deviate from parity structure: \( S_T = S_0 \exp[(\pi - \pi^* - b)T] \), where \( b \) represents the (magnitude of the) rate of depreciation of foreign currency caused by the government’s foreign exchange interventions. Accordingly, the foreign currency nominal price of a F-Bond and the associated time \( T \) real taxes are respectively:\(^{26}\)

\[
B_{F,0} = \exp(-r^*F T) = \exp[-(\rho^* + \pi^* + b^eF)T]
\]
\[
\tau_{F,T} = (D_0 / P_0) \exp [(\rho^* + b^eF - b)T]
\]

where \( b^eF \) is the rationally anticipated magnitude of downward bias or managed depreciation in the exchange rate when the government issues F-Bonds and \( r^* \) is the corresponding foreign currency borrowing rate for the home government. This foreign currency borrowing rate potentially differs from the exogenous borrowing rate, \( r^* \), of the foreign government when \( b^eF \) is non-zero. For the sake of brevity, we shall refer to \( b \) as the foreign exchange bias from hereon. Traditionally this bias has been assumed to be zero in the credibility literature.

When the government issues F-Bonds alone, the real financing benefits of unanticipated foreign exchange interventions are to be weighed against the costs of such interventions in maximizing social welfare. The government chooses the actual levels of \( \pi \) and \( b \) by maximizing the following social welfare function:

\[
U = \alpha (\pi - \pi_{ef}^*) - 0.5 \delta \pi^2 - 0.5 \phi b^2 - w \ln \left\{ (D_0 / P_0) \exp [(\rho^* + b^eF - b)T] \right\}
\]

In keeping with the literature, we assume that the costs of foreign exchange bias as seen by the society are quadratic in the actual rates of foreign exchange bias. Also, the

---

\(^{26}\) To keep our focus on the policy issues, in this paper we work with minimal stochastic modeling. We consider stochastic variations only in the exchange rate. In reality, foreign exchange interventions, if any, will be stochastic in nature to merge with other sources of stochastic variation in the exchange rate.
society views zero foreign exchange bias as desirable. The weight $\phi$ represents the social welfare importance of the costs of foreign exchange bias. The costs of foreign exchange interventions include transaction costs, costs of reputation loss and potential terms of trade effects. The benefits are lower real taxes and the consequent availability of real financing caused by unanticipated depreciation of the foreign currency for any given rate of inflation.

The government’s optimal choice of inflation rate and foreign exchange bias are given by:

$$\pi^F = \frac{\alpha}{\delta}$$  \hspace{1cm} (26)

$$b^F = \frac{wT}{\phi}$$  \hspace{1cm} (27)

While the financing related inflation bias ($wT/\delta$) of H-Bonds is eliminated, it has been effectively replaced by a foreign exchange bias ($wT/\phi$) of F-Bonds. It is important in this context to evaluate the suitability of F-Bonds considering the net impact on social welfare or utility. The F-Bonds leads to an additional bias, namely the foreign exchange bias, and thus an additional source of negative impact on social welfare. The suitability of F-Bonds then depends on the relative importance of the costs of foreign exchange bias and inflation bias to the society.

**Proposition 2:**

If $\phi > \delta$, i.e., if the society views the costs of foreign exchange bias to be more important than the costs of inflation, (and the government can alter the exchange rate through foreign exchange intervention without affecting inflation), then the foreign currency denominated bond is a better choice than the home currency denominated bond.

Proof: With the rationally anticipated inflation and foreign exchange biases inserted into the objective function, the real taxes are $(D \rho \rho) \exp[\rho^T T]$ regardless of the choice of bonds. Let $\phi = q\delta$. Also add a cross-product term to the maximized objective function for the foreign currency bond to complete the square,\(^{27}\) so that the maximized values are equal for $q=1$. Then the maximized values of the objective function for the home currency and the F-

---

\(^{27}\) In optimization, we ignore the cross-product term as it involves the product of inflation and foreign exchange bias and as such is small in magnitude.
Bonds are respectively \(-0.5[\alpha+wT]/\delta\) and \(-0.5[\alpha+(wT/\sqrt{q})]/\delta\). It can be seen now that the maximized value of the objective function is higher for the foreign currency denominated bond if \(q>1\), i.e., when \(\phi>\delta\).

A reciprocal argument of course shows the preference of H-Bonds when the society is more concerned about the inflation bias than the foreign exchange bias, i.e., when \(\phi<\delta\). Intuitively, this is because whichever bias is deemed more important ends up having a relatively lower optimized value and hence when squared reduces the social loss.

3.2 Portfolio of bonds and the exchange rate credibility problem

Let us now revisit the portfolio of bonds solution in light of the exchange rate credibility problem. As before the government sells packages of one H-Bond and one F-Bond per package. Each H-Bond has a face value of \(K\) units of home currency while each F-Bond has a face value of one unit of foreign currency.

With exchange rate intervention possibility, the market’s rational expectation of the time \(T\) exchange rate \(E_M(S_T)\), the current market value proportion of financing in terms of H-Bonds \(m_P\), the (government’s) expected time \(T\) exchange rate \(E_G(S_T)\), the proportion of expected real taxes that the government needs to pay off the H-Bonds \(m_{\tau b}\), and the (government’s) expected time \(T\) real taxes \(E_G(\tau_{PT})\) are respectively:

\[
E_M(S_T) = S_0 \exp\{\left(\pi^eP - \pi^* - b\right)T\}\]  
(28)

\[
m_P = K / \{K + E_M(S_T)\}\]  
(29)

\[
E_G(S_T) = S_0 \exp\{\left(\pi - \pi^* - b\right)T\}\]  
(30)

\[
m_{\tau b} = K / \{K + E_G(S_T)\}\]  
(31)

\[
E_G(\tau_{PT}) = \{m_P D_0 / P_0\} \exp[\left(\rho^* + \pi^eP - \pi\right)T] / m_{\tau b}\]  
(32)

In equation (32) for the government expected real taxes, \((m_P D_0 / P_0)\) is the total real refinancing from H-Bonds at time 0 and \((\rho^* + \pi^eP - \pi)\) is the government expected real cost of borrowing for H-Bonds in continuously compounded terms. Thus, the government expected total real taxes needed to pay off the H-Bonds, \((m_P D_0 / P_0) \exp[\left(\rho^* + \pi^eP - \pi\right)T]\),
combined with the proportion $m_{\tau b}$, leads to the aggregate government expected real taxes $E_G(\tau_{PT})$.

The government chooses the actual level of $\pi$ and $b$ by maximizing the social welfare function:

$$U = \alpha (\pi - \pi^e) - 0.5 \delta \pi^2 - 0.5 \phi b^2 - w \ln \{E_G(\tau_{PT})\}$$  \hspace{1cm} (33)

The first order conditions for this optimization are:

$$\alpha - \delta \pi + wT m_{\tau b} = 0$$  \hspace{1cm} (34)

$$-\phi b + wT (1 - m_{\tau b}) = 0$$  \hspace{1cm} (35)

In the above conditions, $\delta \pi$ is the marginal (welfare) cost of an inflation while $(\alpha + wT m_{\tau b})$ is the associated marginal benefit. For the foreign exchange bias, the marginal cost is $\phi b$ and the marginal benefit is $wT (1 - m_{\tau b})$, that is proportional to the proportion of expected real taxes that the government will need to pay off the F-Bonds.

Note that the proportion $m_{\tau b}$ is determined by the difference between the government’s inflation and foreign exchange bias choices, $\pi - b$, and not by the individual levels of these biases. If we may call this difference the net inflation bias, an increase in the net inflation bias leads to a fall in $m_{\tau b}$ due to the expected appreciation of the foreign currency obligation and hence diminishes (enhances) the marginal real financing benefit of inflation (foreign exchange bias). This is because the proportion of the inflation-sensitive home currency component of the payment obligation shrinks. The opposite effect takes place when the net inflation bias decreases. In other words, with foreign exchange interventions, the government has two choices for gaining real financing and output flexibility that are equivalent in this respect at the margin.

The net inflation bias reflects the preference of the inflation route as opposed to the foreign exchange bias route to create real financing flexibility. This preference is primarily determined by the importance ($\delta$ versus $\phi$) that the society places on the costs of the inflation bias relative to the costs of the foreign exchange bias. The level of the biases, on the other hand, are influenced by the size of the marginal social benefit parameters ($\alpha$, $w$ and $T$) relative to the marginal social cost parameters ($\delta$ and $\phi$).
Proposition 3:
In a dependent central bank framework when the government entertains both inflation ($\pi$) and (negative) exchange rate biases ($b$), uses a portfolio of home currency and foreign currency bonds for financing, and the financing terms are given, then (i) the net inflation bias, defined as $\pi - b$, varies directly with $q$, where $q(=\phi/\delta)$ indicates the importance of the costs of foreign exchange bias relative to the costs of the inflation bias to the society; and (ii) the level of the biases $\pi$ and $b$ increases with $\alpha$, $w$ and $T$.

A formal proof of this proposition is not possible as the solutions for $\pi$ and $b$ can only be obtained numerically given the bond terms. We, therefore, provide an intuitive explanation. The intuition behind (i) is that with a lower $q$ meaning a greater emphasis on the costs of the inflation bias, the marginal social costs of inflation increases relative to that of foreign exchange bias and hence makes the foreign exchange bias a more desirable route to obtain flexibility. The intuition behind (ii) is that the marginal benefits of output expansion and lower real taxes or more real financing increase with $\alpha$, $w$ and $T$. Hence, the government can optimally afford to pursue higher inflation and foreign exchange biases. It is a bit intriguing that when the productivity is higher and the society is more averse to the costs of real taxes, the policy credibility problems are more acute. The effect of bond maturity on inflation credibility was noted by Missale and Blanchard (1994). Here we find the same result in the context of inflation plus exchange rate credibility problems.

Proposition 3 implies that the relative valuation and hence the financing mix (in market value terms) of home currency and F-Bonds will vary according to $q$.

Corollary 2:
In a dependent central bank framework when the government entertains both inflation ($\pi$) and (negative) exchange rate biases ($b$), uses a portfolio of home currency and F-Bonds for financing, and the financing terms are given, then the financing mix in market value terms tilts towards the home (foreign) currency denominated bonds with a lower $q(=\phi/\delta)$. 

If the costs of inflation bias are deemed more important, i.e., if \( q \) is lower, and hence the net inflation bias is lower, the valuation of the H-Bonds relative to the F-Bonds will be higher. The reason is that the biases lead to higher nominal interest rates and lower prices for both types of bonds via rational expectation of inflation and foreign exchange intervention effect. However, as the inflation bias decreases relative to the foreign exchange bias, the H-Bonds are affected less adversely. Therefore, with a lower \( q \), the market value proportion of H-Bonds will tend to be higher. This does not necessarily mean that the proportion of H-Bonds will be lower than the proportion of F-Bonds in an absolute sense as the absolute proportions depend on other factors as well including the face values and the spot exchange rate.

It is worth noting that in this paper the financing mix between the home and F-Bonds is determined endogenously once the inflation and foreign exchange biases are decided upon and given the terms of the home and F-Bonds. It is more traditional in the literature to treat the choice of the financing mix in an independent manner.

Proposition 3 and Corollary 2 constitutes an explanation of the empirical cross-country variation in financing mix reported by Falcetti and Missale (2000). While their explanation is based on a conservative independent central bank and maximum use of H-Bonds, the explanation here is rooted in the dual credibility problem in a dependent central bank framework and the relative importance of inflation and exchange rate biases in terms of social welfare. In this paper, an important factor behind the choice of government bond denomination in various countries is the opportunity to effectively intervene in the foreign exchange market and the emphasis the citizenship places on the social welfare costs of foreign exchange intervention vis-à-vis inflation.

If the economy is large and the home currency is used as a reserve currency by others, the opportunity to effectively intervene on a sustained basis will be limited (hence less flexibility in deriving benefits from interventions). Governments in these large and developed economies also tend to be more concerned about the costs of inflation than the costs of foreign exchange interventions (a lower \( q \)) and about the need for output flexibility. Among other factors, this may explain why the inflation rate tends to be low in countries like the US, Germany, UK and Japan and they do not use as much foreign currency bonds and rely so heavily on home currency bonds. Also, given the large pool of capital stock in these rich countries, the real labor productivity in these countries is likely lower relative to that in
the small and emerging economies. Therefore, the marginal benefit of an inflation surprise in the form of output expansion is likely lower in the capital-rich countries.

In contrast, in smaller European economies and many emerging economies elsewhere with small currency markets, the government is a major player in these markets and often practices adjustable pegging. Therefore, sustained foreign exchange interventions are more doable in these economies. In general, citizens in these economies also tend to be more tolerant of the costs of inflation (low $\delta$) and output fluctuations (do not value output flexibility as much). While their tolerance for foreign exchange interventions may be high (low $\phi$) as well, these countries are also more dependent on foreign capital and foreign trade. Aware of the negative terms of trade effects and foreign capital rationing effects, the governments in these countries are likely to place relatively more emphasis on the costs of foreign exchange intervention (low but higher $\phi$) than their counterparts in the large and developed economies. As a result, the small and emerging economies are likely to find foreign currency denominated financing relatively more desirable than the large and developed economies. Our results here thus provide an alternative explanation of Falcetti and Missale’s (2000) empirical findings about the financing pattern in twenty industrial OECD countries.

One potential danger of relying on F-Bonds and prolonging a foreign exchange bias is that the government’s ability to launch and sustain interventions may weaken. This could happen if the exchange rate regime switches to a free float and/or deteriorating economic fundamentals and repeated intervention lead to dwindling reserves. As shown by the 1997-1998 currency crisis, when this happens, the economic consequences can be fatal.

Interestingly, the foreign exchange bias can be reduced if a highly (weight) conservative independent agency conducts the foreign exchange policy. In fact, if the independent agency is more conservative about the foreign exchange bias than the central banker is about inflation, the F-Bonds may be a better alternative than the home currency bond in an institutional-cum financial solution along the line of Falcetti and Missale (2000). However, in reality, such a mechanism appears more remote than an independent monetary authority in small and emerging economies.

---

28 In fact, many emerging economies are known for keeping the value of their currency at higher levels than dictated by parity for long spans of time.
3.3 The exchange rate credibility problem and the zero inflation policy

Similar to the case of no foreign exchange intervention, a zero rate of inflation can be (optimally) achieved and thus the inflation bias can be totally eliminated when there is an exchange rate credibility problem. This happens if the government selects \( K < 0 \). The specific choice of \( K \) is given in the following proposition:

**Corollary 3:**

In a dependent central bank framework when the government entertains both inflation \((\pi)\) and (negative) exchange rate biases \((b)\), uses a financing package of one K face value home currency bond and one unit face value foreign currency bond, and the financing terms are given (not chosen optimally), then a zero rate of inflation can be achieved by setting:

\[
K = - \left[ \frac{\alpha}{(\alpha + wT)} \right] S_0 \exp[(\pi - \pi^* - b)T] \tag{36}
\]

**Proof:** Inserting the above value of \( K \) in the first order condition for the inflation rate, the condition reduces to \( \delta \pi = 0 \) and hence the solution \( \pi = 0 \) obtains.

The effect of a zero inflation policy on the foreign exchange bias is given in the corollary below.

**Corollary 4:**

In a dependent central bank framework when the government practices a zero inflation \((\pi = 0)\) policy but entertains (negative) exchange rate bias \((b)\), the optimal foreign exchange bias is:

\[
b = \frac{(\alpha + wT)}{\phi} \tag{37}
\]
Proof: Inserting the zero inflation value of $K = - \frac{\alpha}{\alpha + wT} S_0 \exp[(\pi - \pi^* - b)T] \) in the first order condition for the exchange rate bias (b), the condition reduces to $\alpha + wT - \phi b = 0$ and hence the solution $b = (\alpha + wT) / \phi$ obtains.

This zero inflation choice means issuing more F-Bonds than is needed to meet the financing need and then lend the excess in home currency to the private sector. Not only this type of government financing is uncommon in light of real life experience, a zero inflation financing policy can prove to be worse to the society than using H-Bonds alone (and experience maximum inflation bias).

Corollary 5:

In a dependent central bank framework when the government practices a zero inflation ($\pi = 0$) policy but entertains (negative) exchange rate bias (b), the society is worse off using a portfolio of bonds than issuing H-Bonds alone if $\phi < \delta$.

Proof: When the government uses H-Bonds alone, the optimal inflation rate is given by $\pi = (\alpha + wT) / \delta$ and the maximized social welfare is $-0.5 (\alpha + wT)^2 / \delta$. With the zero inflation policy and the portfolio of bonds, the maximized social welfare is $-0.5 (\alpha + wT)^2 / \phi$. When $\phi < \delta$, we have $-0.5 (\alpha + wT)^2 / \phi < -0.5 (\alpha + wT)^2 / \delta$.

Once again it is shown here that countries that are more concerned about the inflation bias than the exchange rate bias, are less likely to issue F-Bonds in their pursuit of minimizing the inflation credibility problem.

To summarize, in a dependent central bank framework, a discretionary government using a financing mix of home and foreign currency bonds may entertain inflation surprise as well as (negative) foreign exchange surprise to obtain real financing and output flexibility. In that case, an exchange rate credibility problem arises in addition to the well-known inflation credibility problem. Thus the gain in inflation credibility from the use of foreign currency bonds is reduced. However, flexibility is enhanced as the foreign exchange bias constitutes a second source of discretionary financing. A critical factor behind the choice of the inflation versus foreign exchange bias to obtain the flexibility of discretionary financing is the
emphasis the society places on the costs of foreign exchange bias relative to the costs of inflation.

4. Optimal Bond Mix

We have thus far derived the optimal ex-post inflation and foreign exchange bias policies as (nonlinear) functions of the bond terms that includes the face value $K$ for the home currency bond and one unit of foreign currency for the foreign currency bond. The purpose of this section is to explore the ex-ante optimal choice of $K$ given the ex-post inflation and foreign exchange bias policies as functions of $K$ and other parameters ($\theta$: $\rho^*, \pi^*, T, S_0, \alpha, \delta, w, \phi, \sigma$).

It is perhaps worthwhile to go over the sequence of events and decisions before we discuss the optimal choice of $K$ (and the bond mix). At time 0, first the government announces the financing terms (the package and its terms, mainly $K$), and then the agents form their rational expectations $\pi^{eq}$ and $b^{eq}$, and accordingly enter into $T$-period nominal wage contracts and purchase the financing package at a price reflecting $\pi^{eq}$ and $b^{eq}$. Given $\pi^{eq}$ and $b^{eq}$, the government selects constant optimal levels of per period actual inflation $\pi$ and foreign exchange bias $b$.

The government (as well as the agents) knows this sequence of events and decisions ahead of time. Therefore, there is no reason to expect that the government will not choose $K$ optimally as well before announcing the financing package as $K$ is a major determinant of the ex-post optimal levels of inflation and foreign exchange bias. It is, however, important to recognize the sequential two-step nature of the government’s overall optimization problem.

First, the optimal ex-post or actual inflation and foreign exchange bias policies ($\pi(\theta, K), b(\theta, K)$) are derived as functions of $K$ and $\theta$ by maximizing $U(\pi, b; \theta, K, \pi^{eq}, b^{eq})$ with respect to $\pi$ and $b$:

$$U(\pi, b; \theta, K, \pi^{eq}, b^{eq}) = \alpha(\pi - \pi^{eq}) - 0.5 \delta \pi^2 - 0.5 \phi b^2 - w \ln\{E_G(\tau_{PT})\}$$

(38)

where, to repeat,

$$E_G(\tau_{PT}) = \frac{m_{e/}}{m_{c/}} \frac{(D_{f/} / P_0)}{\exp[(\rho^* + \pi^{eq} - \pi)T]}$$

(39)

and
m_p = K / \{K + E_M(S_T)\} \quad \text{(40)}
\[m_{\tau b} = \frac{K}{\{K + E_G(S_T)\}}\] (41)

E_M(S_T) = S_0 \exp\{(\pi^{ep} - \pi^* - b^{ep})T\} \quad \text{(42)}
E_G(S_T) = S_0 \exp\{(\pi - \pi^* - b)T\} \quad \text{(43)}

To repeat, the first order conditions for this optimization are:
\[
\frac{\partial U(\theta,K,\pi,b)}{\partial \pi} = \alpha - \delta \pi + w^T m_{\tau b} = 0 \quad \text{(44)}
\]
\[
\frac{\partial U(\theta,K,\pi,b)}{\partial b} = -\phi b + w^T (1-m_{\tau b}) = 0 \quad \text{(45)}
\]

In this process, the government treats \(\pi^{ep}\) and \(b^{ep}\) as given but recognizes that the chosen (solution to (44) and (45) for given bond terms) inflation and foreign exchange bias policies are the ones the agents will use in conjunction with the bond terms (K) in forming their rational expectations of \(\pi\) and \(b\).

Second, using \(\pi = \pi(\theta,K) = \pi^{ep}\) and \(b = b(\theta,K) = b^{ep}\) in \(U(\pi,b,\theta,K,\pi^{ep},b^{ep})\) leads to:
\[
U(\pi(\theta,K),b(\theta,K)) = -0.5 \delta (\pi(\theta,K))^2 - 0.5 \phi (b(\theta,K))^2 -(D_{\pi}P_0) \exp[\rho^*T] \quad \text{(46)}
\]

The government then maximizes \(U(\pi(\theta,K),b(\theta,K))\) with respect to \(K\)\(^{29}\). Using (44) and (45) in (46) in the spirit of the Envelope Theorem, the first-order condition for this optimization is:
\[
\frac{\partial U(\pi(\theta,K),b(\theta,K))}{\partial K} = 0
\]
\[
\{- (\alpha + w^T m_{\tau b})wT/\delta \} + \{(1-m_{\tau b})(wT)^2/\phi \} \{(m_{\tau b}/K)^2S_0 \exp[(\pi - \pi^* - b)T]\} = 0 \quad \text{(47)}
\]

Operationally, there are three equations (44, 45 and 47) to solve for the three unknowns \(\pi\), \(b\) and \(K\). Since \((m_{\tau b}/K)^2S_0 \exp[(\pi - \pi^* - b)T]\) ≠ 0, condition (47), \(\partial U(\theta,K)/\partial K = 0\), reduces to:
\[
(\alpha + w^T m_{\tau b})/\delta = (1-m_{\tau b})(wT)/\phi \quad \text{(48)}
\]

\(^{29}\) The Envelope Theorem does not quite apply here. Here the maximum value function \(U(\theta,K)\) uses \(\pi = \pi(\theta,K) = \pi^{ep}\) and \(b = b(\theta,K) = b^{ep}\) and not just \(\pi = \pi(\theta,K)\) and \(b = b(\theta,K)\). The government optimizes with respect to \(K\) before the bond terms are announced. At that time, the government does not know the agents' expectation of \(\pi\) and \(b\) and hence cannot take them as given, although for the purpose of determining the ex-post optimal inflation and foreign exchange bias policies, the government knows that it would have the knowledge of the agents' expectations. In optimizing with respect to \(K\), the government then needs to assume that the agents' yet unknown rational expectations would be \(\pi^{ep} = \pi(\theta,K)\) and \(b^{ep} = b(\theta,K)\). A notable implication of this is that the government's optimization of \(K\) rests only on the weighted costs of inflation and foreign exchange biases.
Solving equations (44), (45) and (48) leads to ex-ante optimal inflation and foreign exchange biases and the optimal face value of the H-Bonds. Given the importance of these results, we formally state them in the following lemma.

Lemma 1:

In a dependent central bank framework, suppose:

(i) that the discretionary risk-neutral government of a small economy uses a portfolio of T-maturity zero coupon home currency bonds each with a face value of K units of home currency and T-maturity zero coupon foreign currency bonds each with a face value of one unit of foreign currency to refinance an existing amount of debt $D_0$ in home currency;

(ii) that the nominal exchange rate follows the risk-neutral Geometric Brownian Motion:

$$\frac{dS}{S} = (r - \bar{r}) \, dt + \sigma \, dW$$

where $r = \rho^* + \pi^e$ is the interest rate on home currency bond, $r^* = \rho^* + \pi^* + b^*$, $\rho^*$ and $\pi^*$ are the exogenous foreign real rate and inflation rate, and $\pi^e$ and $b^e$ are the rational expectations of the home inflation rate and the downward bias in the exchange rate due to the discretionary government’s intervention in the foreign exchange market; and

(iii) that the government maximizes the social welfare function

$$U = \alpha (\pi - \pi^e) - 0.5 \, \delta \pi^2 - 0.5 \, \phi \, b^2 - w \ln E_G(\tau_T)$$

with respect to the inflation rate $\pi$ and the foreign exchange bias $b$ first and then with respect to the face value $K$ of home currency bonds, where $E_G(\tau_T)$ is the government’s expectation of real taxes needed at time $T$ to payoff the bonds.

Then, the ex-ante optimal inflation and foreign exchange biases and the optimal face value of home currency bonds are respectively:

$$\pi(\theta) = (\alpha + wT) / (\delta + \phi) \quad (49)$$
$$b(\theta) = (\alpha + wT) / (\delta + \phi) \quad (50)$$
Some key policy aspects of Lemma 1 are now stated in the following corollaries.\(^{30}\) First let us characterize the ex-ante optimal inflation and foreign exchange biases.

**Corollary 6:**
The ex-ante optimal inflation rate \(\pi\) and the ex-ante optimal foreign exchange bias \(b\) are positive, that is, there exists a dual credibility problem, namely an inflation credibility problem and an exchange rate credibility problem.

**Corollary 7:**
The ex-ante optimal levels of \(\pi\) and \(b\) are higher if the marginal benefits of discretion (flexibility benefits), \(\alpha\) and \(wT\), are greater relative to the marginal social costs of inflation and foreign exchange bias (credibility costs), \(\delta\) and \(\phi\).

**Corollary 8:**
The ex-ante optimal inflation rate \(\pi\) and the ex-ante optimal foreign exchange bias \(b\) are equal and thus the ex-ante net inflation bias \(\pi-b\) is zero.

Corollary 7 that the ex-ante optimal levels of \(\pi\) and \(b\) are higher if \(\alpha\) and \(wT\) are greater relative to \(\delta\) and \(\phi\) is the same as in Proposition 3 where the bond terms \((K)\) were given. Corollary 6 that there is an ex-ante dual credibility problem is not surprising either. Corollaries 6 and 7 confirm the previous ex-post optimal results on an ex-ante basis. The distinguishing aspect of selecting the bond terms \((K)\) optimally, as can be seen in Corollary 8, is the neutrality of ex-ante net inflation bias with respect to the society’s relative preference for the costs of inflation and foreign exchange biases. The ex-ante optimal inflation and exchange rate biases are equal, i.e., the ex-ante net inflation bias is zero regardless of \(q(=\phi/\delta)\).

\(^{30}\) As the economic setup is described immediately above, we do not repeat it in the statement of the corollaries.
Previously, in Proposition 3, it was found that for given bond terms, the \textit{ex-post} net inflation bias is lower when q is higher.

The agents form their rational expectations after the government announces the bond terms (K). It is then natural to assume that the \textit{ex-ante} optimal inflation and foreign exchange biases would be the rational expectations of the agents. Therefore, the agents would rationally anticipate a zero net inflation bias. Accordingly, we have:

\[
E_{S_0}(S_T) = E_C(S_T) = S_0 \exp[- \pi^* T] \tag{52}
\]

Using the result in (52) leads to the following lemma about the \textit{ex-ante} optimal bond mix.

**Lemma 2:**

When a discretionary government optimizes bond terms (K) in addition to selecting the optimal inflation and exchange rate bias policies, the \textit{ex-ante} optimal proportion of home currency bond financing is:

\[
m_p = [1 - \{\phi / (\delta + \phi)\}] - [(\alpha / wT)\{\phi / (\delta + \phi)\}] \tag{53}
\]

Notice that the optimal bond mix (via the \textit{ex-ante} optimal K) is critically dependent on the society’s relative preference for the costs of inflation and exchange rate biases. This relative preference is indicated by \(\phi / (\delta + \phi)\) or stated alternatively by \(q/(1+q)\), where \(q = \phi / \delta\) as defined earlier. Since \(q/(1+q)\) is increasing in \(q\), the relative preference is effectively summarized by \(q\).

Based on Lemma 2, we can now characterize the nature of optimal bond mix. Once again given the importance of this characterization, we formally state the policy implications in the form of corollaries below.

**Corollary 9:**

Other exogenous parameters remaining the same, the \textit{ex-ante} optimal proportion of home (foreign) currency bond financing is lower (higher) when q is higher, that is the society is relatively more concerned about the costs of foreign exchange bias than the costs of inflation bias.
Corollary 10:
Other exogenous parameters remaining the same, the ex-ante optimal proportion of home (foreign) currency bond financing is lower (higher) when:
(a) $\alpha$ is higher, that is the marginal output benefit from an inflation surprise is greater;
(b) $w$ is lower, that is, the society is not as concerned about the costs of real taxes; and
(c) $T$ is lower, that is the common length of policy horizon and bond maturity is shorter.

Corollary 9 and Corollary 10 may help explain the empirically observed pattern of government financing. As discussed earlier, the small and emerging economies are likely to be characterized by a higher $q$ than the large and developed economies. The former group is also expected to have higher $\alpha$ (marginal product of labor in a Cobb-Douglas economy) due to the low capital and output base. Further, we may expect a lower $w$ in the small and emerging economies compared to the large and developed economies as the citizens of the former group tend to be less sensitive to tax burden. Long-term labor contracts are not as prevalent either in the small and emerging economies leading to a lower $T$. Political instability, higher default risk (resulting in deep discounts for longer maturity bonds) and cultural biases towards cash vis-a-vis long-term securities may further contribute to a shorter policy horizon and bond maturity in the small and emerging economies.

Based on these features and our theoretical results in Corollary 9 and 10, we would expect the small and emerging economies to use relatively more foreign currency bond financing than the large and developed economies. This foreign financing bias in the small and emerging economies or alternatively the home financing bias in the large and developed economies is of course a salient feature of the empirically observed financing pattern.

Our next result in Corollary 11 is about the pursuit of zero inflation in the generalized dual credibility framework when the bond terms are optimized ex-ante. Previously, in Corollaries 3 and 4, we showed that when the bond terms ($K$) are set to achieve zero inflation, the government needs to issue more F-Bonds than is required to meet the financing need and then lend the excess in home currency to the private sector. Then Corollary 5 showed that the society is worse off using this financing policy than issuing H-Bonds alone (and experience maximum inflation bias) if $\phi < \delta$. 
Corollary 11:
If a discretionary government entertains foreign exchange bias but optimizes the bond terms (K), a zero inflation policy is never optimal.

From equation (49), when the government optimizes bond terms and follows ex-post optimal policies, the optimal ex-ante inflation is always positive as long as $\alpha$ and $w$ are positive. The intuition is that the ex-ante optimization of bond terms assumes that the government will use discretion in setting the ex-post optimal inflation rate (and foreign exchange bias) instead of sticking to a zero inflation policy.

5. Conclusion

In this paper, we have generalized the monetary policy setup to allow the possibility that a discretionary government may be able and willing to intervene in the foreign exchange market to bias the exchange rate in its favor when foreign currency denominated bonds are issued. By and large the credibility literature has so far adopted the Relative Purchasing Power Parity as a maintained hypothesis. In the generalized framework of this paper, we have shown that it is ex-post optimal for the government to create a downward bias in the exchange rate in addition to the well-known inflation bias. We then analyzed the nature of the inflation and exchange rate biases and the optimal mixture of home and foreign currency bond financing in the context of the generalized dual credibility model. Essentially, the use of a portfolio of (home and foreign currency) bonds creates a portfolio of instruments or biases to pursue output expansion or to obtain real financing by lowering real taxes. This extended flexibility benefit is to be traded off against the costs of the inflation and foreign exchange biases as viewed by the society.

The predictions of our generalized dual credibility model are in line with the empirically observed pattern of government financing, most recently reported by Falcetti and Missale (2000). They find that despite their inflation bias, the conventional fixed rate home currency bonds are very popular. At the same time, governments have in general reduced their reliance on foreign currency bonds although they help control inflation temptations. Cross-sectionally, however, there is a dichotomy. The relative reliance on home currency
bonds is much heavier in the large and developed economies and the foreign currency bonds remain more popular in the smaller economies.

According to the dual credibility model, when the bond terms are not optimized at the time of issuance, the inflation bias is relatively smaller than the foreign exchange bias if the society is more concerned about the costs of inflation than the costs of foreign exchange bias. In terms of market value, this will enhance the proportion of home currency bond financing. When the government optimizes the bond terms, here the optimal bond mix via the choice of the home currency bond’s face value, the society’s relative preference for the costs of inflation and foreign exchange biases create a similar home currency bias in the optimal mix of home and foreign currency bonds. This type of financing pattern is typically found in the large and developed economies. By the same reasoning, the dual credibility model predicts a relatively higher inflation bias and a lesser home currency bias in the small and emerging economies where it is easier for the government to intervene in the foreign exchange market and the society is more concerned about the foreign exchange bias.

In the dual credibility model, the level of the inflation and foreign exchange biases increase if the flexibility benefits of discretion are greater relative to the credibility costs. This helps explain why small and emerging economies tend to have higher inflation and why the governments in these economies are more active in managing the exchange rate. These economies have of course a much lower capital base than the large economies and their citizens are typically less tolerant of taxes and are taxed less.

In addition to explaining the cross-sectional pattern of policy biases and financing mix, the generalized dual credibility problem also contributes towards understanding the worldwide trend towards more home currency financing, lower inflation and less foreign exchange intervention. In the model, the inflation and foreign exchange biases will drift down if increasing concern about the social costs of government discretion are not matched by increase in productivity and social concern for the costs of real taxes. The increasing home bias in government financing is perhaps driven by that fact that the social costs of inflation (relative to the flexibility benefits) has become more important than the costs of foreign exchange bias. The unpleasant inflationary experience of the seventies and eighties might have triggered and perpetuated this trend.

Falcetti and Missale’s (2000) solution of independent and conservative central banker and maximal home currency financing also addresses many of the empirical patterns in
policy biases and public debt financing mentioned above. Their model adopts the Relative Purchasing Power Parity (RPPP) as a maintained hypothesis. The generalized dual credibility model of this paper provides an alternative explanation that is instead rooted in systematic deviations from the RPPP engineered by a discretionary government. While much has been written about foreign exchange intervention, its role in the context of credibility versus flexibility debate remained obscure. This paper highlights the crucial role of foreign exchange intervention in a policy optimization context and suggests that an optimal level of intervention (if possible at all) in conjunction with an optimal use of foreign currency bonds may actually help attain socially desirable goals. This is particularly so for countries in transition toward truly independent (conservative) monetary institutions.

We conclude with some limitations of the generalized dual credibility framework and the need for further research. The model in this paper suggests optimal ex-ante inflation and exchange rate biases to be equal. The level of foreign exchange intervention required for this may be difficult to implement for many smaller countries with low reserves or for countries where the impact of intervention on money supply and thus inflation may be material. There is also the possibility that foreign exchange interventions that bias the exchange rate downwards may ultimately create a negative terms of trade effect for export-dependent countries. The effect of the foreign exchange interventions on the conditional volatility of the exchange rate needs to be addressed as well.
REFERENCES


Barro, Robert J. and Gordon, David B., Rules, Discretion and Reputation in a Model of Monetary Policy, Journal of Monetary Economics, 1983b, 12, 101-121.


